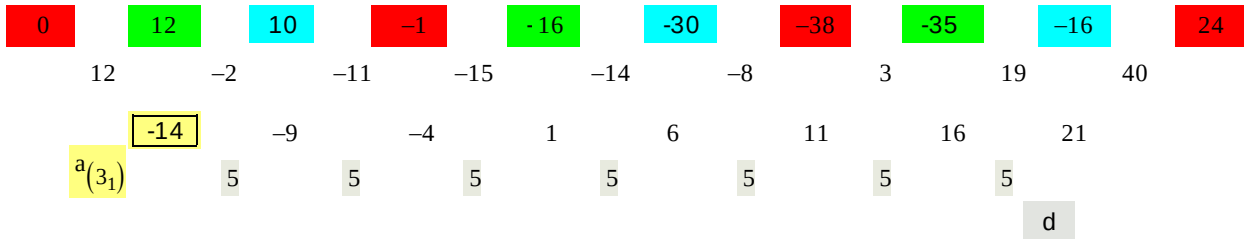
Author: Mohammad Reza Serajian asl

Finding the value of terms in a third difference sequence

There are some methods as (matrix or Cramer's Rule) for solving the third difference sequences problems with some matrix calculations.

By this article we are going to introduce a general formula with easy function and without matrix calculations for determining the values of the terms in third difference sequences.

The mentioned method is based on to this fact, << each one of the third difference sequences can be Divide to three groups {a, b, c} of sequences >> as blow sample.



For determining the values of terms in {a} group we should use of below formula

$$\left(36m^3 - 288m^2 + 770m - 688\right)d + \left(9m^2 - 51m + 72\right)X_a + \left(9m^2 - 45m + 56\right)Y_a + (3m - 7)Z_a + (3m - 8)T_a + a_1 = a_n$$

And for determining the unknown values on it we should use of below formulas.

** The $a_{(3_1)}$ in formulas is the first terms of the third sequences of differences "*the sequences before the common difference*" in above example the value of $a_{(3_1)}$ is, (-14) and the formula for determining the Common difference is $a_2 - 3\cdot c_1 + 3\cdot b_1 - a_1 = d$ for above example it will be $-1 - 3\cdot 10 + 3\cdot 12 - 0 = 5$

$\frac{n+4}{2} = m$ $\quad a_{(3_1)} + \frac{d}{2} = X_a$ $\quad a_{(3_1)} - \frac{d}{2} = Y_a$ $\quad b_1 - a_1 = Z_a$ $\quad c_1 - b_1 = T_a$

n = the number of terms, in sequenc $a_1, a_2, a_3, \ldots, a_n\}$

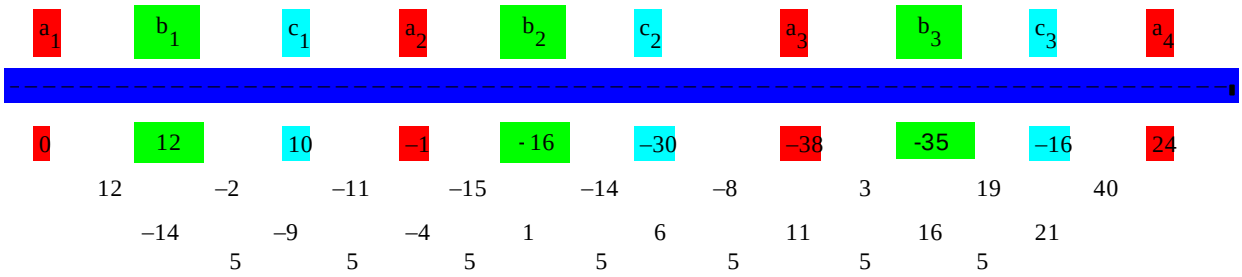
a_n = the values of the terms in sequenc $a_1, a_2, a_3, \ldots, a_n\}$

For example: the third term of {a} group in above sequence is $\{a_n = a_3\}$ and the value of it will be obtain via below math expression

$$\left(36\times 3.5^3-288\times 3.5^2+770\times 3.5-688\right)5+\left(9\times 3.5^2-51\times 3.5+72\right)\cdot -11.5+\left(9\times 3.5^2-45\times 3.5+56\right)\cdot -16.5+(3\times 3.5-7)12+(3\times 3.5-8)\cdot -2+0=-38$$

For another example: the fourth term of $\{a\}$ group in above sequence is $\{a_n = a_4\}$and the value of it will be obtain via below math expression

$$\left(36\times 4^3-288\times 4^2+770\times 4-688\right)5+\left(9\times 4^2-51\times 4+72\right)\cdot -11.5+\left(9\times 4^2-45\times 4+56\right)\cdot -16.5+(3\times 4-7)12+(3\times 4-8)\cdot -2+0=24$$



For determining the values of terms in $\{b\}$ group we should use of below formula

$$\left(36m^3-288m^2+770m-688\right)d+\left(9m^2-51m+72\right)X_b+\left(9m^2-45m+56\right)Y_b+(3m-7)Z_b+(3m-8)T_b+b_1=b_n$$

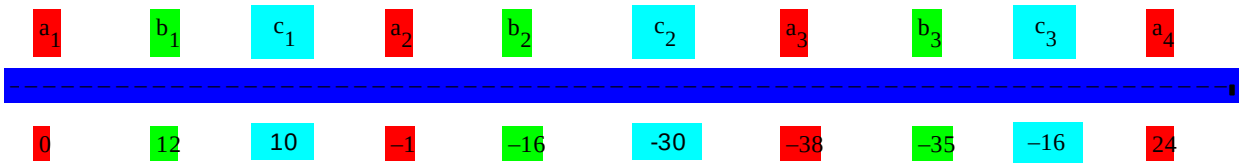
And for determining the unknown values on it we should use of below formulas

common difference}= d $\quad \frac{n+4}{2}=m \quad a_{(3_1)}+\frac{3d}{2}=X_b \quad a_{(3_1)}+\frac{d}{2}=Y_b \quad c_1-b_1=Z_b \quad a_2-c_1=T_b$

n = the number of terms, in sequenc $b_1, b_2, b_3, \ldots, b_n\}$ $\quad$ b_n = the values of the terms in sequenc $b_1, b_2, b_3, \ldots, b_n\}$

For example: the third term of $\{b\}$ group in above sequence is $\{b_n = b_3\}$and the value of it will be obtain via below math expression

$$\left(36\times 3.5^3-288\times 3.5^2+770\times 3.5-688\right)5+\left(9\times 3.5^2-51\times 3.5+72\right)\cdot -6.5+\left(9\times 3.5^2-45\times 3.5+56\right)\cdot -11.5+(3\times 3.5-7)\cdot -2+(3\times 3.5-8)\cdot -11+12=-35$$



For determining the values of terms in $\{c\}$ group we should use of below formula

$$\left(36m^3-288m^2+770m-688\right)d+\left(9m^2-51m+72\right)X_c+\left(9m^2-45m+56\right)Y_c+(3m-7)Z_c+(3m-8)T_c+c_1=c_n$$

And for determining the unknown values on it we should use of below formulas

common difference = d $\frac{n+4}{2} = m$ $a_{(3_1)} + \frac{5d}{2} = X_c$ $a_{(3_1)} + \frac{3d}{2} = Y_c$ $a_2 - c_1 = Z_c$ $b_2 - a_2 = T_c$

n = the number of terms, in sequenc $c_1, c_2, c_3, \ldots, c_n\}$ c_n = the values of the terms in sequenc $c_1, c_2, c_3, \ldots, c_n\}$

For example: the third term of $\{b\}$ group in above sequence is $\{b_n = b_3\}$ and the value of it will be obtain via below math expression

common difference = d $\frac{n+4}{2} = m$ $a_{(3_1)} + \frac{5d}{2} = X_c$ $a_{(3_1)} + \frac{3d}{2} = Y_c$ $a_2 - c_1 = Z_c$ $b_2 - a_2 = T_c$

n = the number of terms, in sequenc $c_1, c_2, c_3, \ldots, c_n\}$ c_n = the values of the terms in sequenc $c_1, c_2, c_3, \ldots, c_n\}$

$\frac{3+4}{2} = 3.5$ n = 3 d = 5 $a_{(3_1)} = -14$ $-14 + \frac{5 \times 5}{2} = -1.5$ $-14 + \frac{3 \times 5}{2} = -6.5$ $-1 - 10 = -11$ $-16 - (-1) = -15$

$(36 \times 3.5^3 - 288 \times 3.5^2 + 770 \times 3.5 - 688)5 + (9 \times 3.5^2 - 51 \times 3.5 + 72) \cdot -1.5 + (9 \times 3.5^2 - 45 \times 3.5 + 56) \cdot -6.5 + (3 \times 3.5 - 7) \cdot -11 + (3 \times 3.5 - 8) \cdot -15 + 10 = -16$

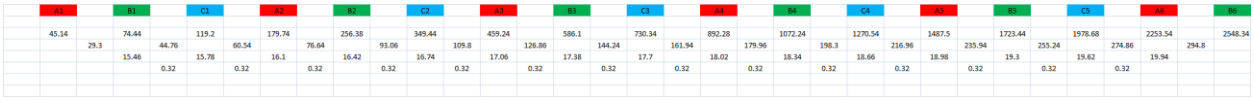
Key words: Third difference sequences; sequences;

Below is some related formula for above article.

$(36 \times 4^3 - 288 \times 4^2 + 770 \times 4 - 688)5 + (9 \times 4^2 - 51 \times 4 + 72) \cdot -11.5 + (9 \times 4^2 - 45 \times 4 + 56) \cdot -16.5 + (3 \times 4 - 7)12 + (3 \times 4 - 8) \cdot -2 + 0 = 24$

$(36 \times 3.5^3 - 288 \times 3.5^2 + 770 \times 3.5 - 688)5 + (9 \times 3.5^2 - 51 \times 3.5 + 72) \cdot -6.5 + (9 \times 3.5^2 - 45 \times 3.5 + 56) \cdot -11.5 + (3 \times 3.5 - 7) \cdot -2 + (3 \times 3.5 - 8) \cdot -11 + 12 = -35$

$(36 \times 3.5^3 - 288 \times 3.5^2 + 770 \times 3.5 - 688)5 + (9 \times 3.5^2 - 51 \times 3.5 + 72) \cdot -1.5 + (9 \times 3.5^2 - 45 \times 3.5 + 56) \cdot -6.5 + (3 \times 3.5 - 7) \cdot -11 + (3 \times 3.5 - 8) \cdot -15 + 10 = -16$



$(36 \times 5^3 - 288 \times 5^2 + 770 \times 5 - 688)0.32 + (9 \times 5^2 - 51 \times 5 + 72)15.62 + (9 \times 5^2 - 45 \times 5 + 56)15.3 + (3 \times 5 - 7)29.3 + (3 \times 5 - 8)44.76 + 45.14 = 2253.54$

$(36 \times 5^3 - 288 \times 5^2 + 770 \times 5 - 688)0.32 + (9 \times 5^2 - 51 \times 5 + 72)15.94 + (9 \times 5^2 - 45 \times 5 + 56)15.62 + (3 \times 5 - 7)44.76 + (3 \times 5 - 8)60.54 + 74.44 = 2548.34$

$(36 \times 4^3 - 288 \times 4^2 + 770 \times 4 - 688)0.32 + (9 \times 4^2 - 51 \times 4 + 72)16.26 + (9 \times 4^2 - 45 \times 4 + 56)15.94 + (3 \times 4 - 7)60.54 + (3 \times 4 - 8)76.64 + 119.2 = 1270.54$

$a_2 - 3 \cdot c_1 + 3 \cdot b_1 - a_1 = d$ $179.74 - 3 \cdot 119.2 + 3 \cdot 74.44 - 45.14 = 0.32$

Author: Mohammad Reza Serajian asl

serajianasl@yahoo.com

$15.46+\frac{0.32}{2}=15.62$

$15.46-\frac{0.32}{2}=15.3$

$15.46+\frac{3\cdot 0.32}{2}=15.94$

$15.46+\frac{0.32}{2}=15.62$

$15.46+\frac{5\cdot 0.32}{2}=16.26$

$15.46+\frac{3\cdot 0.32}{2}=15.94$

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